Quantitative Reasoning over Incomplete Abstract Argumentation Frameworks

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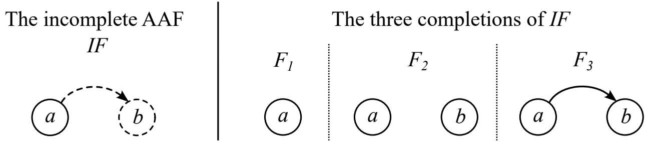
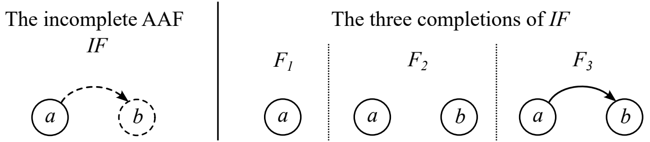
A brief review of Incomplete Abstract Argumentation Frameworks

- *Incomplete Abstract Argumentation Frameworks* (iAAFs) are AAFs where arguments and attacks can be marked as *uncertain* (as their occurrence in the argumentation graph is not guaranteed)
- The uncertainty is modeled qualitatively: no measure of the extent of the uncertainty is encoded (differently from quantitative approaches, such as *probabilistic AAFs*, *weighted AAFs*, etc.)
- An iAAFs compactly represents a set of alternative configurations of the argumentation graph, called *completions*



A brief review of Incomplete Abstract Argumentation Frameworks

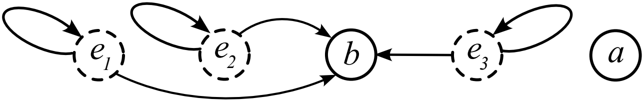
- The reasoning is based on the verification and acceptance problems under the possible and necessary perspective:
  - A set is a possible (resp., necessary) i*-extension if it is an extension in at least one (resp., every) completion;
  - An argument is possibly (resp., necessarily) accepted if it is accepted in at least one (resp., every) completion.
- ***Example.*** Under $\sigma = \mathrm{co}$, the set $\{a\}$ is a possible (but not necessary) i*-extension. *a* is necessarily skeptically accepted, *b* is possibly skeptically accepted.



Motivation: Limits of the reasoning paradigm

- The possible and necessary perspectives are “extreme" and may hide relevant issues

- ***Example.*** Under $\sigma = \text{co}$, both $S_1 = \{a\}$ and $S_2 = \{a, b\}$ are possible (but not necessary) i*-extensions. Indeed, S_1 is extension in 7 out of 8 completions (those where at least one among e_1, e_2, e_3 occurs) while S_2 in only 1 out of 8 completions (the completion where e_1, e_2, e_3 do not occur)



- If we replaced the possible and necessary perspective with a mechanism for counting the completions where a set is an extension, S_1 and S_2 would be no more indistinguishable!

Contribution: A quantitative reasoning paradigm based on counting the completions

- **General (and challenging) scenario:** a set $\mathcal{D}$ of dependencies between arguments/attacks is specified to discard “unrealistic" completions
  - e.g. $\textsc{or}(a, b)$, $\textsc{nand}\Big((a, b), (b, a)\Big)$, $\textsc{choice}(a, b)$, etc.

- We introduce ***a new quantitative reasoning paradigm*** based on the problems:
  - $\textsc{PercVer}^{\sigma}(IF, \mathcal{D}, S)$: Return the percentage of completions of the iAAF *IF* satisfying $\mathcal{D}$ where S is an extension;
  - $\textsc{PercAcc}^{\sigma}(IF, \mathcal{D}, a, X)$: Return the percentage of completions of the iAAF *IF* satisfying $\mathcal{D}$ where a is X-accepted ($X \in \{$credulously, skeptically$\}$);
  - $\textsc{cntCom}(IF, \mathcal{D})$: Return the number of completions of the iAAF *IF* satisfying $\mathcal{D}$.

Contribution 1: Relationship with probabilistic AAFs

- prAAFs (under the constellations approach) are iAAFs where a probability distribution function is defined over the completions
- Fundamental problems over prAAFs:
  - PROBVER and PROBACC: return the overall probability of the completions where S is an extension and a is accepted, respectively.
- ***QUESTION:*** Isn't it straightforward to encode the iAAF IF as a prAAF PF such that PROBVER$^\sigma(PF, S)$=PERCVER$^\sigma(IF, \mathcal{D}, S)$ and PROBACC$^\sigma(PF, a, X)$=PERCACC$^\sigma(IF, \mathcal{D}, a, X)$?
- ***ANSWER:*** Surprisingly... NO! This would require an exponential-time computation.
  - ***Hint:*** defining the pdf of the prAAF requires knowing the number of completions, and we prove that CNTCOM is #P-complete

Contribution 1: Relationship with probabilistic AAFs

- ***QUESTION:*** Are there cases where the translation from iAAFs to prAAFs (for the purpose of solving PERCVER and PERCACC) is convenient (i.e. feasible in polynomial time)?
- ***Answer:*** Yes! This happens when both the following hold:
  - $\mathcal{D}=\emptyset$;
  - every uncertain attack in *IF* involves at least one certain argument
- The case above generalizes the cases:
  - only arguments can be uncertain
  - only attacks can be uncertain

Contribution 2: Complexity characterization

- **General result:** PERCVER and PERCACC are $FP^{\#P}$-complete, and CNTCOM is #P-complete, even if $\mathcal{D} = \emptyset$;
  - CNTCOM's hardness proved via a reduction from the problem of evaluating the overall weight of the homomorphisms between a graph and a weighted graph;
  - PERCVER's and PERCACC's hardness proved via a reduction from CNTCOM;

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- **Tractability island**: CNTCOM is in P if $\mathcal{D} = \emptyset$ and every uncertain attack involves at least one certain argument (the same form allowing easy translatability from iAAFs to prAAFs)
- ***QUESTION:*** Can we generalize this tractability result to the case $\mathcal{D} \neq \emptyset$?
- ***ANSWER:*** Unfortunately, NO! If $\mathcal{D}$ contains some dependency (even of only one form among OR, NAND, CHOICE, IMPLY), even if in *IF* the uncertainty involves only arguments or only attacks, CNTCOM is #P-complete, and PERCVER and PERCACC are $FP^{\#P}$-complete.

Future work

- Extending the framework, towards the definition of the core of a complex analysis cockpit
  - Simultaneously look into the counts of completions and extensions
  - Answer questions like: "*How many sets of arguments are extensions in at least 80% of the completions?*

Thank you!