Ignorance is not Bliss:

Deontic Risk and the Asking of Questions

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Motivation

To model *evocation* of questions bearing on *normative issues*, when reasoning under *uncertainty*.

- uncertainty – are conditional norms triggered? are defeaters available?
- normative issues – obligations $\Rightarrow$ actions; not always possible to suspend belief
- question evocation – connects to zetetic epistemology; conservation of cognitive resources and directed inquiry are key.

Outline

Conceptual Background

DACQ - The Deontic Argumentation Calculus with Questions

Argumentation

Correspondance Results

Conclusion

Evoked Questions Norm

EQN: Rationality requires that you have an *Inquiring Attitude* toward a *Question* only if

(1) Your knowledge entails that the *Question* has a true, direct answer, and

(2) Your knowledge does not entail any direct answer to the *Question*.

- We focus on (2); (1) would require a richer logical language to adequately capture.

Willard-Kyle, C., Millson, J., & Whitcomb, D. (2024). Evoked questions and inquiring attitudes. The Philosophical Quarterly. *pp. 384–406*

Our Four Rationality Postulates (1/2)

Affirmative openness. Don't ask what you already know.

Deontic relevance. Ask only if the answer could change what you ought to do.

- *Openness:* you know $p \vee q$ and have $p \rightsquigarrow k$, $q \rightsquigarrow k$; so k is already fixed — asking whether p holds is a "doh question".[1]
- *Relevance:* you know p and $p \rightsquigarrow k$, but not h; since $k \wedge h \rightsquigarrow b$, h can change your duty.

[1] Here: $x \rightsquigarrow y$: informal conditional norm, "if x, then (ought) y".

Our Four Rationality Postulates (2/2)

Defeasibility sensitivity. Check potential defeaters before acting.

Minimality. Ask the least specific question that resolves the issue. (See paper)

- *Defeasibility:* you know p and $p \rightsquigarrow k$, but not l; since $l \rightsquigarrow \neg k$, acting on k now risks a hidden contrary duty.
- *Minimality:* you don't know p or q but have $p \rightsquigarrow k$, $q \rightsquigarrow k$; asking $p \vee q$ suffices, so asking p alone is overly specific.

The Calculus

Deontic Argumentation Calculi

$$\mathsf{AX}\ \frac{\vdash_{\mathbf{C}} \Gamma \Rightarrow \Delta}{\Gamma^x \Rightarrow \Delta^x} \qquad \mathsf{FDet}\ \frac{}{\phi^f, (\phi, \psi)^n \Rightarrow \psi^o}$$

$$\mathsf{DDet}\ \frac{\phi^f, \Gamma \Rightarrow \Delta}{\phi^o, \Gamma \Rightarrow \Delta} \qquad \mathsf{Def}\ \frac{\Gamma, (\phi, \psi)^n \Rightarrow}{\Gamma \Rightarrow \neg(\phi, \psi)^n}$$

$$\mathsf{Cut}\ \frac{\Gamma \Rightarrow \phi \qquad \phi, \Gamma' \Rightarrow \Delta}{\Gamma, \Gamma' \Rightarrow \Delta} \qquad \mathsf{OR}\ \frac{\phi_1^f, \Gamma_1 \Rightarrow \psi^o \qquad \phi_2^f, \Gamma_2 \Rightarrow \psi^o}{(\phi_1 \vee \phi_2)^f, \Gamma_1, \Gamma_2 \Rightarrow \psi^o}$$

- $\mathcal{L}^{DAC}$ is a labeled language ($x \in \{f, o, n\}$)
- builds on a base sequent calculus $\mathbf{C}$, here $\mathbf{C} = LK$

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van Berkel, K., & Straßer, C. (2022). Reasoning with and about norms in logical argumentation. In: F. Toni, S. Polberg, R. Booth, M. Caminada, & H. Kido (Eds.), Computational models of argument. *pp. 332–343.*

* For cut-free version see Andrea's talk!

DAC Derivations

Example. A clinical decision system needs to decide whether to prescribe a pain killer. If the patient is in pain the system should prescribe the drug, but if the patient also has a fever the system should not prescribe the drug. Assume that the patient is in pain and has a fever.

$\mathbb{K} = \langle \mathcal{F}, \mathcal{N} \rangle$, for $\mathcal{F} = \{p^f, f^f\}$ and $\mathcal{N} = \{(p,k)^n, (p \wedge f, \neg k)^n\}$.

$$\mathsf{Cut}\ \dfrac{\dfrac{\vdots}{p^f, f^f, (p \wedge f, \neg k)^n \Rightarrow \neg k^o} \qquad \mathsf{Cut}\ \dfrac{\mathsf{FDet}\ \dfrac{}{p^f, (p,k)^n \Rightarrow k^o} \qquad \mathsf{AX}\ \dfrac{\vdash_{\mathbf{C}} k, \neg k \Rightarrow}{k^o, \neg k^o \Rightarrow}}{p^f, (p,k)^n, \neg k^o \Rightarrow}}{\mathsf{Def}\ \dfrac{p^f, f^f, (p,k)^n, (p \wedge f, \neg k)^n \Rightarrow}{p^f, f^f, (p \wedge f, \neg k)^n \Rightarrow \neg (p,k)^n}}$$

Question Evocation Rule 1 (Qnor)

$\phi^{?}_{\mathsf{nor}}$ stands for the normative question:

does ϕ, which entails further obligations, hold?

$$\mathsf{Qnor}\ \frac{\Gamma^{f}, \Delta^{n} \Rightarrow \phi^{o}}{\sim(\bigwedge \Gamma')^{f}, \sim\phi^{o}, (\Gamma \setminus \Gamma')^{f}, \Delta^{n} \Rightarrow \bigwedge \Gamma'^{?}_{\mathsf{nor}}}$$

known fact, obligation, unknown

*for $\emptyset \neq \Gamma' \subseteq \Gamma$

- **defeasible!!** – can be defeated if
  - (a) agent is not ignorant about $\psi \in \Gamma'$ or ϕ^{o}
  - (b) a norm in Δ is unreliable
  - (c) a more minimal question suffices

Derivation with Qnor

Example. The same clinical decision system as before is augmented so that it also recommends monitoring the patient's blood pressure if they take painkillers and have a heart condition; we consider a scenario where the system does not know whether the patient has a heart condition.

$$\mathcal{F} = \{p^f\} \text{ and } \mathcal{N} = \{(p,k)^n (p \wedge f, \neg k)^n, (k \wedge h, b)^n\}$$

$$\dfrac{\dfrac{\dfrac{}{p^f, (p,k)^n \Rightarrow k^o}\,\mathsf{FDet} \qquad \dfrac{\vdots}{k^o, h^f, (k \wedge h, b)^n \Rightarrow b^o}\,\mathsf{DDet}}{p^f, h^f, (p,k)^n, (k \wedge h, b)^n \Rightarrow b^o}\,\mathsf{Cut}}{\mathbf{a} : {\sim}h^f, {\sim}b^o, p^f, (p,k)^n, (k \wedge h, b)^n \Rightarrow h^?_{\mathsf{nor}}}\,\mathsf{Qnor}$$

Question Evocation Rule 2 (Qdia)

$\phi^{?}_{\mathsf{dia}}$ stands for the dialectic question:

does ϕ, which entails the defeat of a committed norm, hold?

$$\mathsf{Qdia}\ \frac{\Gamma_1^f, \Delta_1^n \Rightarrow \neg(\phi,\psi)^n \qquad \Gamma_2^f, \Delta_2^n, (\phi,\psi)^n \Rightarrow \psi^o}{\sim(\bigwedge\Gamma'), \sim\neg(\phi,\psi)^n, (\Gamma_1 \setminus \Gamma')^f, \Gamma_2^f, (\Delta_1 \cup \Delta_2)^n \Rightarrow (\bigwedge\Gamma')^{?}_{\mathsf{dia}}}$$

*for $\Gamma' \in \wp(\Gamma_1) \setminus \{\emptyset\}$

- **relates to deontic risk** – acting on ψ^o without checking whether Γ' could lead to a norm violation

$$\dfrac{\dfrac{\vdots}{p^f, f^f, (p \wedge f, \neg k)^n \Rightarrow \neg(p,k)^n} \qquad \dfrac{}{p^f, (p,k)^n \Rightarrow k^o}\,\mathsf{FDet}}{\sim f^f, \sim\neg(p,k)^n, p^f, (p \wedge f, \neg k)^n \Rightarrow f^{?}_{\mathsf{dia}}}\,\mathsf{Qdia}$$

Ignorance Rules (1/2)

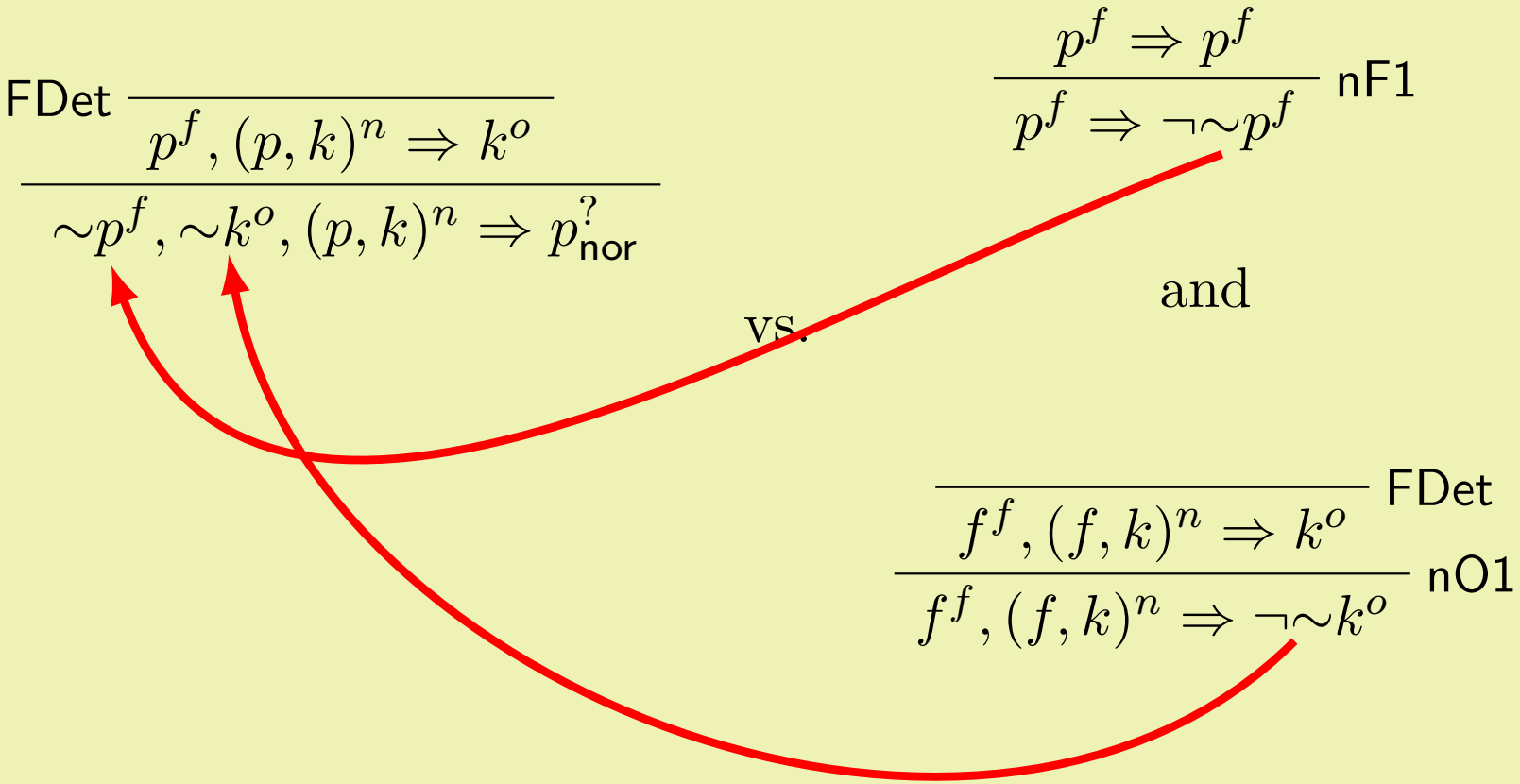
If you can derive ϕ^x, you are not ignorant about ϕ^x!

$$\mathsf{nF1}\ \frac{\Gamma^f \Rightarrow \phi^f}{\Gamma^f \Rightarrow \neg\sim\phi^f} \qquad \mathsf{nF2}\ \frac{\Gamma^f \Rightarrow (\neg\phi)^f}{\Gamma^f \Rightarrow \neg\sim\phi^f} \qquad \mathsf{nN}\ \frac{\Gamma^f \Rightarrow \neg(\phi,\psi)^n}{\Gamma^f \Rightarrow \neg\sim\neg(\phi,\psi)^n}$$

$$\mathsf{nO1}\ \frac{\Gamma^f, \Delta^n \Rightarrow \phi^o}{\Gamma^f, \Delta^n \Rightarrow \neg\sim\phi^o} \qquad \mathsf{nO2}\ \frac{\Gamma^f, \Delta^n \Rightarrow (\neg\phi)^o}{\Gamma^f, \Delta^n \Rightarrow \neg\sim\phi^o}$$

- **blocking "doh questions"** –
  - with a base calculus where $\vdash_{\mathbf{C}} \phi \Rightarrow \phi$, nF1 and nF2 block questions about known facts
  - all rules block questions about things following from $\mathbb{K}$

Ignorance Rules (2/2)



Argumentation

Dung Semantics

Let $AF = \langle \mathsf{Arg}, \mathsf{Att} \rangle$ be an AF and let $\mathcal{E} \subseteq \mathsf{Arg}$.

- $\mathcal{E}$ is **conflict-free** if there are no $a, b \in \mathcal{E}$ with $(a, b) \in \mathsf{Att}$.
- $\mathcal{E}$ is **stable** if it is conflict-free and attacks every argument not in $\mathcal{E}$.

Given a knowledge base $\mathbb{K}$, a DACQ-induced AF is the pair $\langle \mathsf{Arg}(\mathbb{K}), \mathsf{Att}(\mathbb{K}) \rangle$, where

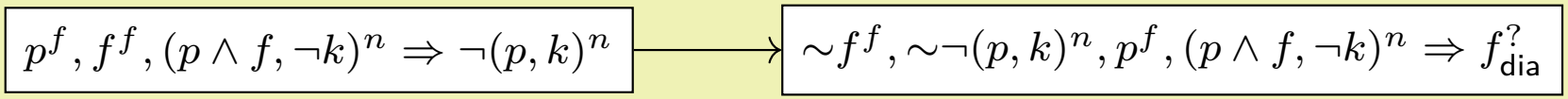
- $\mathsf{Arg}(\mathbb{K})$ is all $\mathbb{K}$-based DACQ derivable arguments
- $\mathsf{Att}(\mathbb{K})$ are ...

Dung, P. M. (1995). On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games. Artificial Intelligence. 77, *pp. 321–357.*

Attacks in Detail

- **Norm attacks.** $\Gamma \Rightarrow \neg(\phi, \psi)^n$ attacks

$$\begin{cases} \Delta, (\phi, \psi)^n \Rightarrow \Theta & \text{Is the norm reliable} \\ \Delta, {\sim}\neg(\phi, \psi)^n \Rightarrow \Theta & \text{Is there really no attack on the norm?} \end{cases}$$



Attacks in Detail

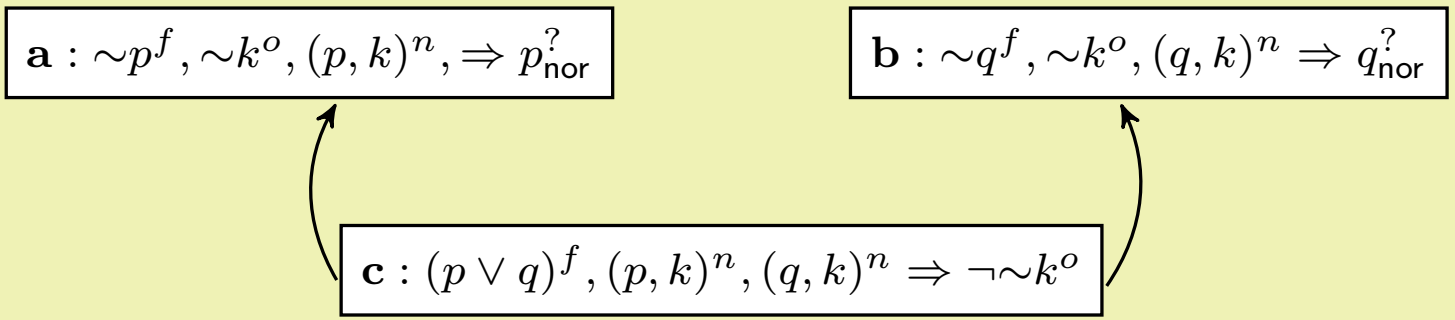
- **Norm attacks.** $\Gamma \Rightarrow \neg(\phi, \psi)^n$ attacks

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$$p^f, f^f, (p \wedge f, \neg k)^n \Rightarrow \neg(p, k)^n \longrightarrow \sim f^f, \sim\neg(p, k)^n, p^f, (p \wedge f, \neg k)^n \Rightarrow f^?_{\mathsf{dia}}$$

- **$\sim f/o$-attacks.** $\Gamma \Rightarrow \neg\sim\gamma^{f/o}$ attacks

$$\begin{cases} \Delta, \sim\gamma^f \Rightarrow \Theta & \text{Are we really ignorant about the fact } \gamma^f \\ \Delta, \sim\gamma^o \Rightarrow \Theta & \text{Are we really ignorant about the obligation } \gamma^o \end{cases}$$



Attacks in Detail

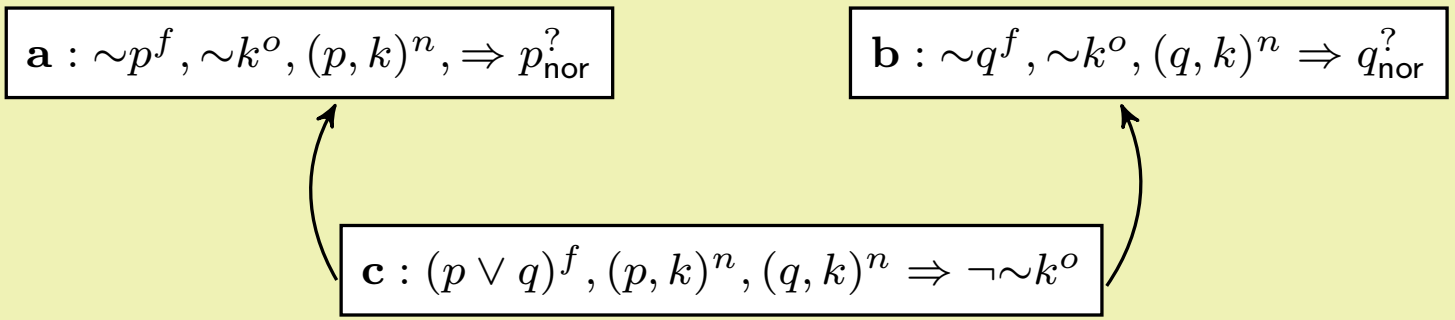
- **Norm attacks.** $\Gamma \Rightarrow \neg(\phi, \psi)^n$ attacks

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$$p^f, f^f, (p \wedge f, \neg k)^n \Rightarrow \neg(p, k)^n \longrightarrow \sim f^f, \sim\neg(p, k)^n, p^f, (p \wedge f, \neg k)^n \Rightarrow f^?_{\mathsf{dia}}$$

- **$\sim f/o$-attacks.** $\Gamma \Rightarrow \neg\sim\gamma^{f/o}$ attacks

$$\begin{cases} \Delta, \sim\gamma^f \Rightarrow \Theta & \text{Are we really ignorant about the fact } \gamma^f \\ \Delta, \sim\gamma^o \Rightarrow \Theta & \text{Are we really ignorant about the obligation } \gamma^o \end{cases}$$



- **Minimality attacks.** see paper.

Suspension in DACQ-induced AFs

For a stable set $\mathcal{E}$, we introduce the following notations:

- $\mathcal{N}^+(\mathcal{E}) = \{(\sigma_1, \sigma_2)^n \in \mathcal{N} \mid \exists \Gamma \Rightarrow \neg(\sigma_1, \sigma_2)^n \in \mathcal{E}\}$
- $\mathbb{K}^{\phi}_{\mathcal{E}} = \langle \mathcal{F} \cup \{\phi^f\}, \mathcal{N} \setminus \mathcal{N}^+(\mathcal{E}) \rangle$

Let $\mathcal{E}$ be a stable extension of $\mathsf{AF}(\mathbb{K})$. We let:

- **(simple validity)** $\mathcal{E} \models \phi$ if there is an argument $\mathbf{a} \in \mathcal{E}$ with $\mathsf{Con}(\mathbf{a}) = \phi$.
- **(suspended obligation, local)** $\mathcal{E} \models^{?} \phi^o$ iff $\mathcal{E} \models \phi^o$ but $\forall \mathbf{a} \in \mathcal{E}$, if $\mathsf{Con}(\mathbf{a}) = \phi^o$ then $\exists \sigma$ with $\mathcal{E} \models \sigma^{?}_{\mathsf{dia}}$ and $\exists \mathbf{b} \in \mathsf{Arg}(\mathbb{K}^{\sigma}_{\mathcal{E}}) \setminus \mathcal{E}$ that norm-attacks $\mathbf{a}$.
- **(all-things-considered obligation, local)** $\mathcal{E} \models^{!} \phi^o$ iff $\mathcal{E} \models \phi^o$ but $\mathcal{E} \not\models^{?} \phi^o$.

Nonmonotonic Inference

We define three skeptical consequene relations:

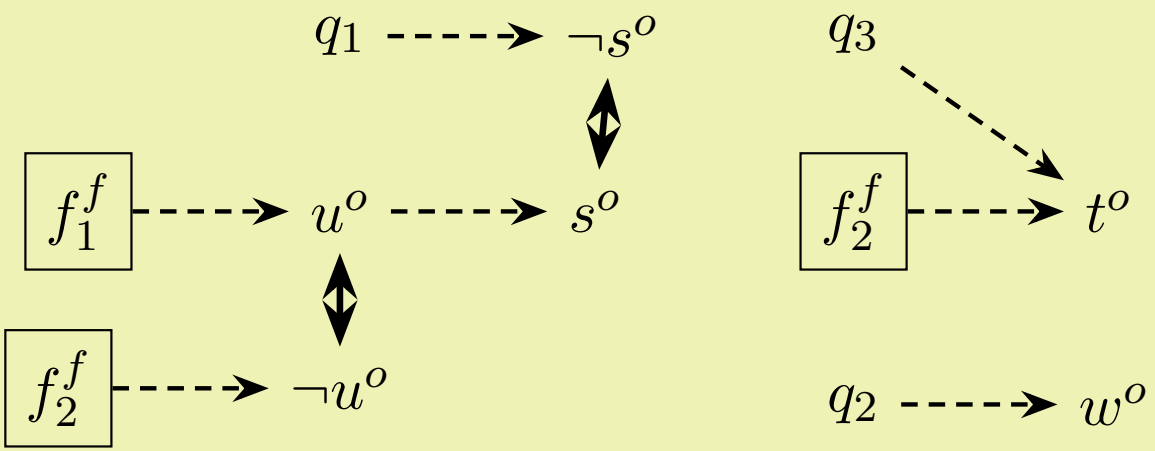
- **(shared conclusions)** $\mathbb{K} \mid\sim \varphi$ iff $\mathcal{E} \models \varphi$ in every stable set $\mathcal{E}$.
- **(all-things-considered obligations, global)** $\mathbb{K} \mid\sim^{!} \varphi^{o}$ iff $\mathcal{E} \models^{!} \varphi^{o}$ in every stable set $\mathcal{E}$.
- **(suspended obligations, global)** $\mathbb{K} \mid\sim^{?} \varphi^{o}$ iff $\mathcal{E} \models \varphi^{o}$ in every stable set $\mathcal{E}$, but $\mathcal{E} \models^{?} \varphi^{o}$ in some stable set.

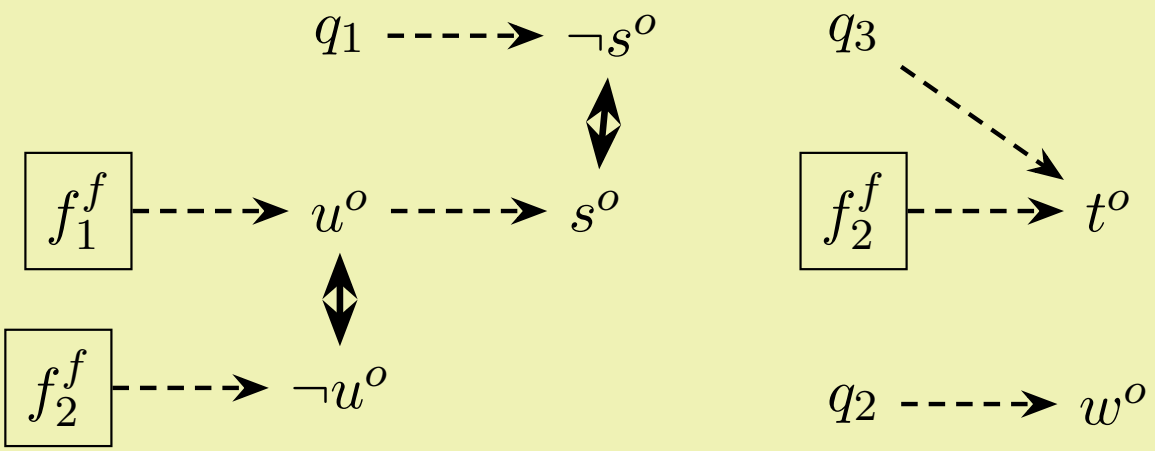
An Example (1/2)

$\mathcal{F} = \{f_1^f, f_2^f\}$, and

$\mathcal{N} = \{(f_1, u)^n, (u, s)^n, (f_2, \neg u)^n, (f_2, t)^n, (q_1, \neg s)^n, (q_2, w)^n, (q_3, t)^n\}$.

Schematic:





$\mathbf{a_u} : f_1^f, (f_1, u)^n \Rightarrow u^o, \qquad \mathbf{a_s} : f_1^f, (f_1, u)^n, (u, s)^n \Rightarrow s^o,$

$\mathbf{a_{nu}} : f_2^f, (f_2, \neg u)^n \Rightarrow (\neg u)^o, \quad \mathbf{a_t} : f_2^f, (f_2, t)^n \Rightarrow t^o$

$\mathbf{b_u}: f_1^f, f_2^f, (f_1, u)^n \Rightarrow \neg(f_2, \neg u)^n, \ \mathbf{b_{nu}}: f_1^f, f_2^f, (f_2, \neg u)^n \Rightarrow \neg(f_1, u)^n$

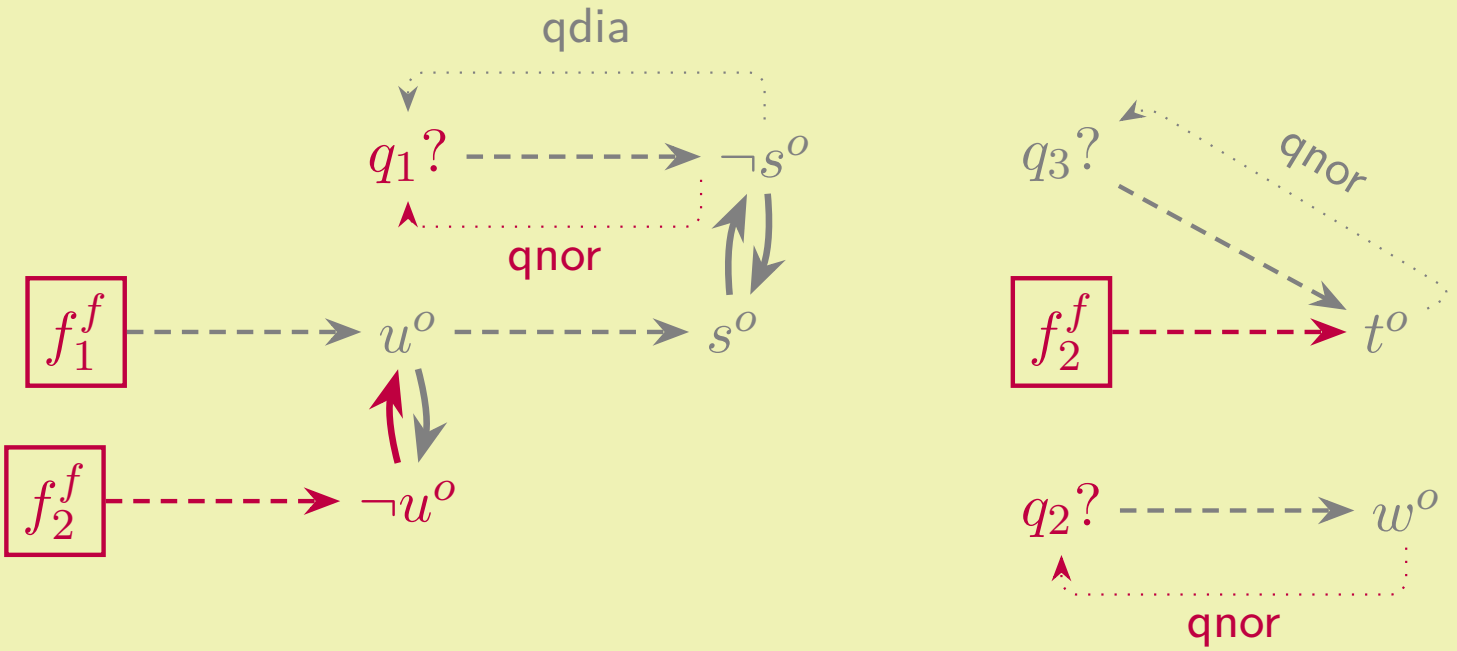
$\mathbf{q_1} : {\sim}q_1^f, {\sim}\neg(u, s)^n, f_1^f, (f_1, u)^n, (q_1, \neg s)^n \Rightarrow {q_1}^?_{\mathsf{dia}},$

$\mathbf{q_{11}} : {\sim}q_1^f, {\sim}(\neg s)^o, (q_1, \neg s)^n \Rightarrow {q_1}^?_{\mathsf{nor}},$

$\mathbf{q_2} : {\sim}q_2^f, {\sim}w^o, (q_2, w)^n \Rightarrow {q_2}^?_{\mathsf{nor}}, \quad \mathbf{q_3} : {\sim}q_3^f, {\sim}t^o, (q_3, t)^n \Rightarrow {q_3}^?_{\mathsf{nor}}$

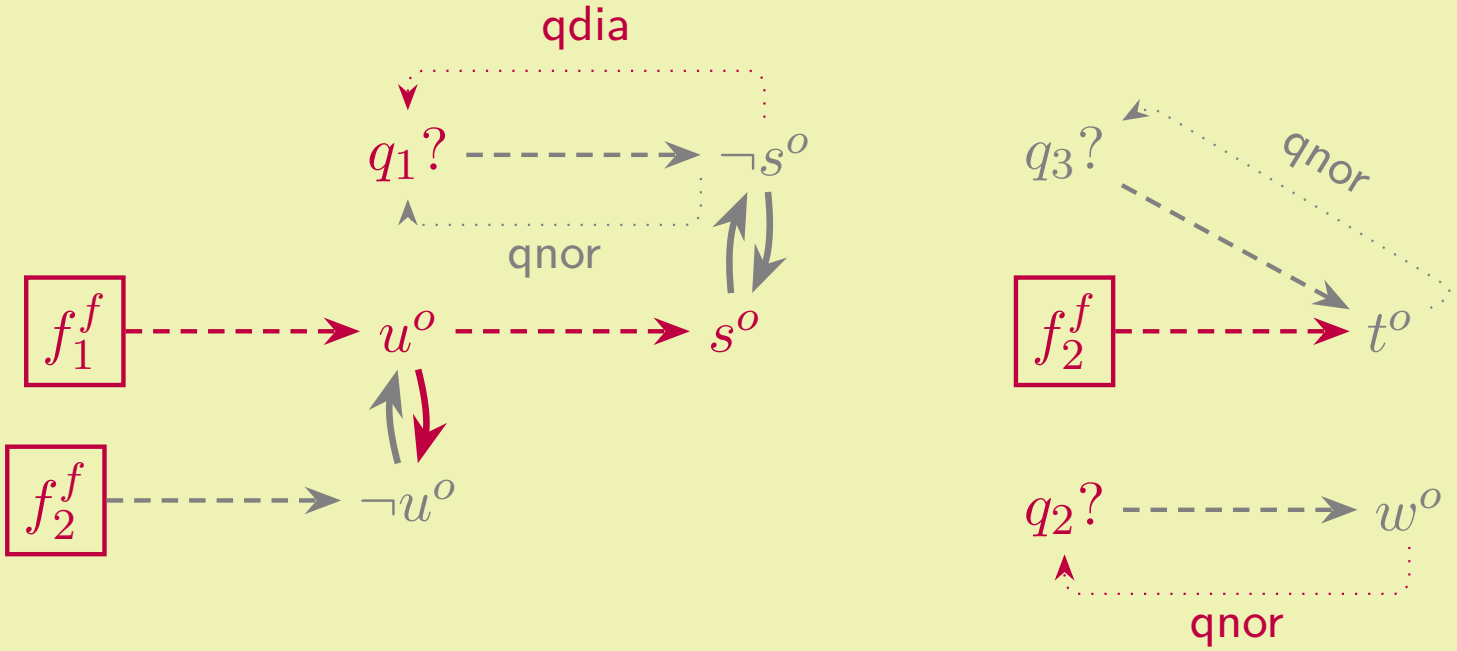
$\mathbf{n_{11}} : f_1^f, (f_1, u)^n, (u, s)^n \Rightarrow \neg{\sim}(\neg s)^o, \ \mathbf{n_3} : f_2^f, (f_2, t)^n \Rightarrow \neg{\sim}t^o$

Stable set $\mathcal{E}_1$



- Obligations: $\mathcal{E}_1 \models \neg u^o$
- Questions: $\mathcal{E}_1 \models q_1^?{}_{\mathsf{nor}}$ and $\mathcal{E}_1 \models q_2^?{}_{\mathsf{nor}}$
- No suspended obligation

Stable set $\mathcal{E}_2$



- Obligations: $\mathcal{E}_2 \models u^o$ and $\mathcal{E}_2 \models s^o$
- Questions: $\mathcal{E}_2 \models q_{1\mathsf{dia}}^?$ and $\mathcal{E}_2 \models q_{2\mathsf{nor}}^?$
- Suspended obligations: $\mathcal{E}_2 \models^? s^o$

Meta-Theory

Theorem 1

Let $\langle \mathsf{Arg}(\mathbb{K}), \mathsf{Att}(\mathbb{K}) \rangle$ be a DACQ-induced AF and let $\mathcal{E} \subseteq \mathsf{Arg}(\mathbb{K})$. Let $\mathcal{E}$ be a stable extension in $\mathsf{AF}(\mathbb{K})$. We have: $\mathcal{E} \models \phi^{?}_{\mathsf{nor}}$ iff

- **(factual ignorance)** $\mathcal{E} \not\models \phi^{f}$ and $\mathcal{E} \not\models (\neg\phi)^{f}$, and
- **(normative relevance)** there is a ψ for which $\mathcal{E} \not\models \psi^{o}$ and $\mathcal{E} \not\models (\neg\psi)^{o}$ while there is an argument $\mathbf{b} \in \mathsf{Arg}(\mathbb{K}^{\phi}_{\mathcal{E}}) \setminus \mathsf{Arg}(\mathbb{K})$ with $\mathsf{Con}(\mathbf{b}) = \psi^{o}$; and for this ψ:
- **(minimality)** there are no ξ and $\mathbf{b} \in \mathsf{Arg}(\mathbb{K}^{\xi}_{\mathcal{E}})$ with $\mathsf{Con}(\mathbf{b}) = \psi^{o}$ and $\phi > \xi$.

Theorem 2

Let $\langle \mathsf{Arg}(\mathbb{K}), \mathsf{Att}(\mathbb{K}) \rangle$ be a DACQ-induced AF and let $\mathcal{E} \subseteq \mathsf{Arg}(\mathbb{K})$. Let $\mathcal{E}$ be a stable extension in $\mathsf{AF}(\mathbb{K})$ and let $\mathbb{K}' = \mathbb{K}^{\phi}_{\mathcal{E}}$. We have $\mathcal{E} \models \phi^{?}_{\mathsf{dia}}$ iff

- **(factual ignorance)** $\mathcal{E} \not\models \phi^{f}$ and $\mathcal{E} \not\models (\neg\phi)^{f}$, and
- **(dialectic relevance)** there are $(\sigma, \psi)^{n} \in \mathcal{N}$ and arguments $\mathbf{a} \in \mathcal{E}$ with $\mathsf{Con}(\mathbf{a}) = \psi^{o}$ and $\mathbf{b} \in \mathsf{Arg}(\mathbb{K}') \setminus \mathcal{E}$, such that $\mathbf{b}$ attacks $\mathbf{a}$ in $(\sigma, \psi)^{n}$; and for this $(\sigma, \psi)^{n}$:
- **(minimality)** there are no ξ and $\mathbf{b} \in \mathsf{Arg}(\mathbb{K}^{\xi}_{\mathcal{E}})$ with $\mathsf{Con}(\mathbf{b}) = \neg(\sigma, \psi)^{n}$ and $\phi > \xi$.

Overview

Our goal: to provide a model of reasoning to normative questions in cases of uncertainty.

- 1^{st} Conceptual background: when are questions rational? extension to normative domain $\Rightarrow$ when do conditional norms recommend inquiry?
- 2^{nd} Logical tools: capturing the desiderata from 1. in a sequent calculus;
- 3^{rd} Argumentation: investigating nonmonotonic inference to questions and obligations & their interaction; norm suspension due to dialectical questions.

Related & Future Work

- Connections to Erotetic/Inquisitive Logics:
  - looking at detailed points of connection
  - considering more model theoretic approaches
- Abduction:
  - acts as a bridge between observations and the belief layer
- Nuance with Norm Conflicts:
  - c.f. [1]; handling specificity, CTDs, etc.
- Planning (see [2])

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[1] – *van Berkel, K., & Straßer, C. (2025). Reasoning with and about normative conflicts. In K. van Berkel, A. Ciabattoni, & J. F. Horty (Eds.)*, Deontic Logic and Normative Systems - 17th International Conference, DEON 2025, *Vienna, Austria, July 1-3, 2025 pp. 21–40. College Publications.*
[2] – *Pardo, P., & Straßer, C. (2024). The goal after tomorrow: Offline goal reasoning with norms.* Journal of Artificial Intelligence Research, *80, pp. 1703–1759*

Appendix

Minimality Rules

If multiple inquiries are directed at the same normative issue, we should prefer the most general question that still resolves the issue.

$$\textsf{MinQnor}\ \frac{\sim\omega_1^f, \sim\phi^o, \Gamma_1^f, \Delta_1^n \Rightarrow \omega_1{}^?_{\textsf{nor}} \qquad \sim\omega_2^f, \sim\phi^o, \Gamma_2^f, \Delta_2^n \Rightarrow \omega_2{}^?_{\textsf{nor}} \qquad \omega_1 > \omega_2}{\sim\omega_2^f, \sim\phi^o, \Gamma_2^f, \Delta_2^n \Rightarrow \neg(\sim\omega_1^f \wedge \sim\phi^o)}$$

$$\textsf{MinQdia}\ \frac{\sim\omega_1^f, \sim\neg(\phi,\psi)^n, \Gamma_1^f, \Delta_1^n \Rightarrow \omega_1{}^?_{\textsf{dia}} \qquad \sim\omega_2^f, \sim\neg(\phi,\psi)^n, \Gamma_2^f, \Delta_2^n \Rightarrow \omega_2{}^?_{\textsf{dia}} \qquad \omega_1 > \omega_2}{\sim\omega_2^f, \sim\neg(\phi,\psi), \Gamma_2^f, \Delta_2^n \Rightarrow \neg((\phi,\psi)^n \wedge \sim\omega_1^f)}$$

*$\omega_1 > \omega_2$ abbreviates $\omega_1 \Rightarrow \omega_2, \omega_2 \not\Rightarrow \omega_1$, i.e. ω_1 is more specific than ω_2

Derivation with MinQnor

Example. Consider a clinical decision system where if the patient is in pain it should prescribe the painkiller and if the patient is queasy it should also prescribe it. This time there is no info about the patient.

$$\mathcal{F} = \{\ \} \text{ and } \mathcal{N} = \{(p,k)^n, (p,q)^n\}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{p^f,(p,k)^n \Rightarrow k^o}\,\text{FDet} \quad \dfrac{}{q^f,(q,k)^n \Rightarrow k^o}\,\text{FDet}}{\mathbf{d} : (p \vee q)^f,(p,k)^n,(q,k)^n \Rightarrow k^o}\,\text{OR}}{\mathbf{a} : {\sim}(p \vee q)^f,{\sim}k^o,(p,k)^n,(q,k)^n \Rightarrow (p \vee q)^?_{\mathsf{nor}}}\,\text{Qnor} \quad \dfrac{\dfrac{}{p^f,(p,k)^n \Rightarrow k^o}\,\text{FDet}}{\mathbf{b} : {\sim}p^f,{\sim}k^o,(p,k)^n \Rightarrow p^?_{\mathsf{nor}}} \quad p > p \vee q}{\mathbf{c} : {\sim}(p \vee q)^f,{\sim}k^o,(p,k)^n,(q,k)^n \Rightarrow \neg({\sim}p^f \wedge {\sim}k^o)}\,\text{MinQnor}$$